

BPT

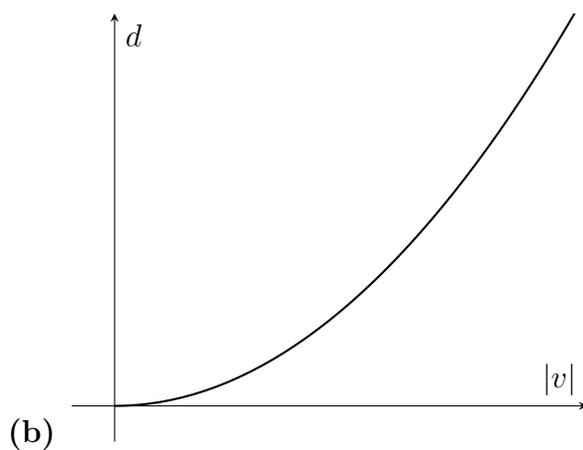
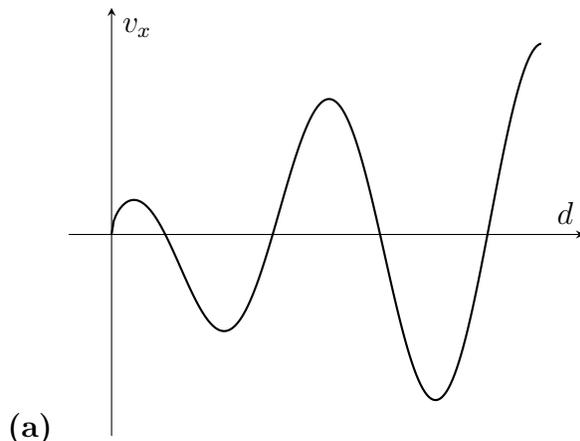
Sample Problems

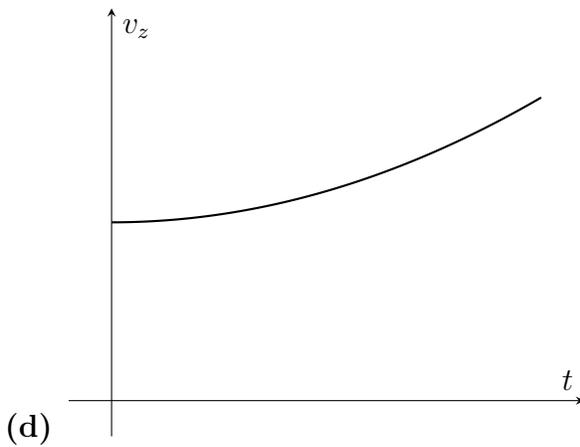
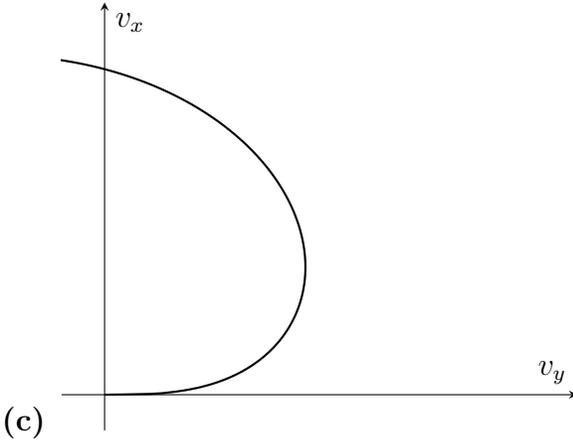
1 Individual MCQ

A ball rolls down a clockwise spiraling slide and experiences 2 revolutions around the slide while traveling downward. Assume no friction or air resistance. Recording begins when the ball lies at

$$(0, r, h)$$

where r is the radius of the slide and h is the height above the ground. Which of these graphs does NOT depict an accurate relationship in this system? v_x and v_y indicate horizontal velocities, v_z indicates vertical velocity, and d denotes the total distance traveled by the ball.





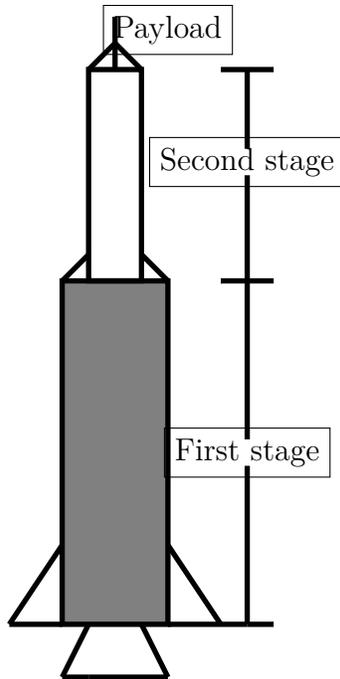
2 Team FRQ

Consider the following simplified two stage rocket. The rocket is delivering a payload, positioned at the top, with mass m . The second stage (directly under the payload) has a mass nm , and the total rocket, including the bottom stage, is Nm ($1 \leq n \leq N$). The fuel is ejected out of the engine at exit velocity of V .

You may use the following formula, commonly known as the rocket equation:

$$\vec{F}_{ext} = M \frac{d\vec{v}}{dt} - \vec{u} \frac{dM}{dt}$$

Where \vec{F}_{ext} is the total external forces acting on the rocket, \vec{v} is the rocket's velocity, M is the rocket's mass, and \vec{u} is the exhaust velocity. Assume the stage separation is instant and exerts no force, and that the mass of the fuel tanks and engines are negligible compared to the mass of fuel.



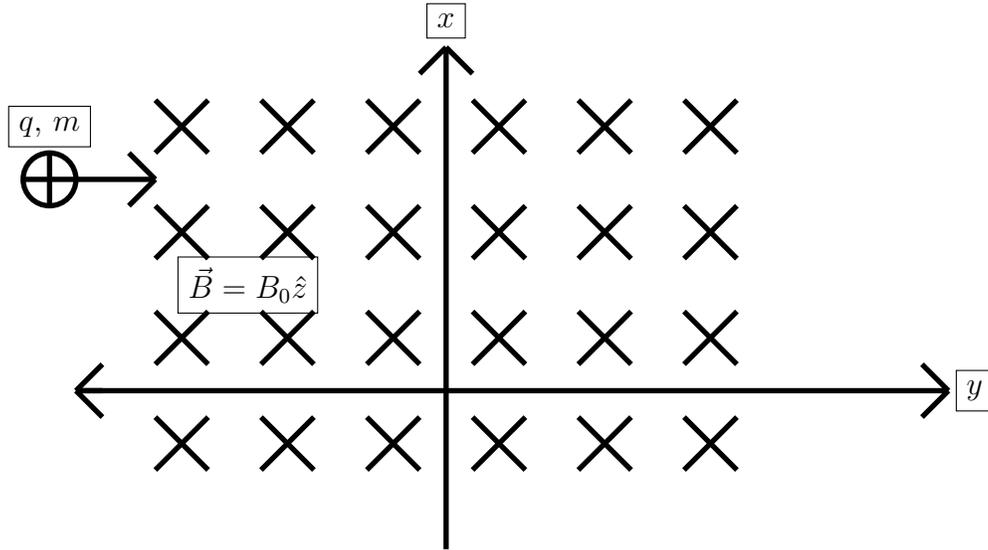
- (a) Consider the two stage rocket in outer space with *no* external forces. What is the change in velocity Δv_1 after burning the first stage?
- (b) After the first stage is released, the second stage starts burning. Show that the final velocity after the second stage finishes burning does not depend on the value of n .
- (c) In a more realistic scenario, the rocket takes off from rest. Consider gravity and air resistance proportional to the velocity given by $\vec{F}_{fr} = -k\vec{v}$ and $\vec{F}_g = -Mg$. What is the maximum change in velocity in this case? Let the gravitational acceleration be g . You may assume the constant of proportionality of the air resistance and the gravitational acceleration does not change as the rocket gains altitude.
- (d) After exiting the atmosphere, the rocket drops the first stage and ignites the second stage. Now, the air is much thinner, so the air resistance is given by $\vec{F}_{fr} = -(k/2)\vec{v}$. At this altitude, you may assume gravity is negligible. What is the change in velocity achievable after the second burn?

3 GUTS

A magician has a loaded hula hoop, which can be modeled as a uniform ring of mass m with a point mass of $m/3$ attached to a point on the ring. The magician places the ring on an incline with a very high friction coefficient, and finds the maximum angle θ in which the hoop can be set on the incline and remain stationary. What is θ ?

4 Individual FRQ

In plasma physics, we consider the collective behavior of charged particles in EM fields. To investigate complicated problems, we usually first consider a simple version of it. Here we consider the motion of a single particle in EM fields.



- (a) With a uniform magnetic field $\vec{B} = B_0 \hat{z}$ in the space, if there is a charged particle with initial velocity $\vec{v} = v_0 \hat{y}$, charge q , and mass m , show that the charged particle will move in a circle, and find the frequency of that circular motion.
- (b) Now if we add another electric field $\vec{E} = E_0 \hat{y}$, how will the motion be different? Try to cancel out the Electric force by considering this in another reference frame.
- (c) Now let's consider the collective behavior: If there are many charged particles like this, with a number density of n (meaning that there are nV particles in a volume of V) show that the setup in (b) gives us a steady current, while (a) doesn't. This is actually a very simple model of Hall Effect, saying that if you put a piece of metal in \vec{E} in y direction and \vec{B} in z direction, there will be a current (or equivalently voltage) in x direction