2 GUTS

GUTS 01.

Knight Launch-a-Lot

Wikipedia: A trebuchet (French: trébuchet) is a type of catapult that uses a hinged arm with a sling attached to the tip to launch a projectile.

(a) A fun fact about trebuchets is that they can launch a 90 kg projectile over 300 meters. Suppose that we have a perfectly ideal trebuchet, which has no mass and no friction, so that all of the counter-weight's gravitational potential energy is converted into the projectile's kinetic energy. Assuming that the launch angle is 45°, and the projectile experiences no air resistance, and the counter-weight falls vertically 4 meters, how heavy must the trebuchet's counter-weight be to launch the projectile at least 300 meters?

(b) In reality, trebuchets are not perfectly efficient. If the counter-weight from part (a) actually launches the projectile only 240 meters (instead of 300 meters) due to energy losses, what is the efficiency of this trebuchet (expressed as a percentage of the ideal case)?

(c) Medieval engineers discovered that adding wheels to the trebuchet frame improves efficiency by allowing the counter-weight to fall more vertically. If adding wheels increases the efficiency to 85% and allows the same counter-weight to fall 5 meters instead of 4 meters, how far will the 90 kg projectile travel now? Assume the launch angle remains 45° and air resistance is still negligible.

Solution

(a) For a projectile launched at speed v and angle 45°, the range is

$$R = \frac{v^2 \sin(2\theta)}{g} = \frac{v^2}{g}$$

To travel 300 m, we need

$$300 = \frac{v^2}{g} \quad \Rightarrow \quad v^2 = 300g$$

The projectile has mass $m = 90$ kg, so its kinetic energy at launch must be

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(90)(300g)$$

If the counter-weight has mass M and falls $h = 4$ m, then its lost gravitational potential energy is

$$Mgh$$

Since the trebuchet is ideal,

$$Mg(4) = \frac{1}{2}(90)(300g)$$

Canceling g,

$$4M = \frac{1}{2}(90)(300) = 13500$$

Thus

$$M = \frac{13500}{4} = 3375 \text{ kg}$$

So the counter-weight must have mass

$$\boxed{3.38 \times 10^3 \text{ kg}}$$

(b) Efficiency is the ratio of the actual output energy to the ideal output energy. Since the launch angle is still 45°, the range is proportional to v^2, and hence proportional to the projectile's kinetic energy. Therefore,

$$\text{efficiency} = \frac{240}{300} = 0.80$$

Expressed as a percentage,

$$\boxed{80\%}$$

(c) We use the same counter-weight from part (a), so $M = 3375$ kg. Now the counter-weight falls 5 m, and the trebuchet is 85% efficient. Thus the projectile's kinetic energy is

$$K = 0.85(Mg \cdot 5)$$

Setting this equal to the projectile's kinetic energy,

$$\frac{1}{2}(90)v^2 = 0.85(3375g)(5)$$

Solve for v^2:

$$v^2 = \frac{2(0.85)(3375)(5)g}{90}$$

Since the launch angle is 45°, the range is

$$R = \frac{v^2}{g} = \frac{2(0.85)(3375)(5)}{90}$$

Compute:

$$R = 318.75 \text{ m}$$

Therefore the projectile will travel

$$\boxed{3.19 \times 10^2 \text{ m}}$$

Answer

(a) 3.38×10^3 kg

(b) 80%

(c) 3.19×10^2 m

GUTS 02.

Caffeinated Collider

(a) You are testing a new "coffee-powered" accelerator in the physics reading room. It launches coffee beans of mass 1 g around a circular tube using compressed air, which applies a constant tangential force of 1 N. Assume the tube forms a circle of radius $r = 0.5$ m and that there is no friction. If the bean starts from rest, what is the angular velocity, ω, in radians per second (to two significant figures) after 10 seconds?

(b) The coffee bean exerts a force on the walls that depends on its speed. If the bean has *velocity* (not *angular velocity*) $v_0 = 10$ m/s, what force do the walls experience, in Newtons to two significant figures?

(c) Once the coffee bean reaches the velocity found in part (a), it is released into a straight pipe and undergoes a perfectly elastic collision with an espresso bean initially at rest. If the espresso bean has mass 0.3 g, how fast is it launched? You may leave your answer as a fraction or as an approximation to three significant figures.

Answer

(a) $\omega = 2.0 \times 10^4$ rad/s

(b) $F_c = 0.20$ N

(c) $v_e = \frac{20}{13} \times 10^4 \text{ m/s} \approx 1.54 \times 10^4 \text{ m/s}$

Solution

(a) First, find the linear acceleration (converting mass to kg):

$$F = ma \implies a = \frac{1\ \text{N}}{10^{-3}\ \text{kg}} = 10^3\ \text{m/s}^2$$
$$\alpha = \frac{a}{r} = \frac{10^3}{0.5} = 2 \times 10^3\ \text{rad/s}^2$$
$$\omega_f = \omega_0 + \alpha t$$
$$= 0 + (2 \times 10^3\ \text{rad/s}^2)(10\ \text{s}) = \boxed{2.0 \times 10^4\ \text{rad/s}}$$

(b) The force on the walls is the normal force, which provides the centripetal force. It can be calculated with the formula:

$$F_c = \frac{mv^2}{r}$$

So, substituting the parameters of the problem:

$$F_c = \frac{(10^{-3}\ \text{kg})(10\ \text{m/s})^2}{0.5\ \text{m}} = \frac{0.1}{0.5} = \boxed{0.20\ \text{N}}$$

(c) The formula for the final velocity of a target mass initially at rest in a perfectly elastic collision is:

$$v_2 = \frac{2m_1}{m_1 + m_2} v_1$$

Since the formula uses a ratio of masses, we can keep the masses in grams:

$$v_e = \frac{2(1\ \text{g})}{1\ \text{g} + 0.3\ \text{g}} v_1 = \frac{2}{1.3} v_1$$

We must find v_1 from the angular velocity in part (a):

$$v_1 = r\omega = (0.5\ \text{m})(2 \times 10^4\ \text{rad/s}) = 10^4\ \text{m/s}$$

Therefore:

$$v_e = \frac{2}{1.3}(10^4\ \text{m/s}) = \boxed{\frac{20}{13} \times 10^4\ \text{m/s} \approx 1.54 \times 10^4\ \text{m/s}}$$

GUTS 03.

Hot Wheels Hullabaloo

(a) A car attempts to perform a loop-the-loop maneuver on a track of radius R, which is much larger than the size of the car. What is the minimum speed required for the car to stay on the track as it crosses the topmost part of the loop if $R = 20$ m? Let $g = 9.8\ \text{m/s}^2$, and express your answer to two significant figures.

(b) Before the car enters the loop, it must first accelerate in a straight line on a flat, horizontal road. If the coefficient of static friction between the tires and the road is $\mu = 0.80$, what is the maximum acceleration the car can achieve in m/s^2 to two significant figures?

(c) Now suppose the car has a mass of 1000 kg and is driving around a flat, circular racetrack with a radius of 100 m. If the car travels at a tangential velocity of 100 m/s, how much centripetal force does the car experience? Express your answer in scientific notation to two significant figures.

Answer

(a) 14 m/s

(b) 7.8 m/s^2

(c) 1.0×10^5 N

Solution

(a) At the top of the loop, gravity and the normal force both point downward toward the center of the circle, providing the centripetal force:

$$N + mg = \frac{mv^2}{R}$$

For the *minimum* speed to stay on the track, the car is on the verge of falling off, meaning the normal force $N \to 0$.

$$mg = \frac{mv^2}{R} \implies v = \sqrt{Rg}$$

$$v = \sqrt{(20)(9.8)} = \sqrt{196} = \boxed{14 \text{ m/s}}$$

(b) The maximum forward force the car can generate without slipping is equal to the maximum static friction, $f_s = \mu N$. On a flat road, $N = mg$. By Newton's Second Law:

$$ma = \mu mg \implies a = \mu g$$

$$a = (0.80)(9.8 \text{ m/s}^2) = 7.84 \text{ m/s}^2$$

Rounding to two significant figures, we get $\boxed{7.8 \text{ m/s}^2}$.

(c) The centripetal force required to keep the car moving in a circle is given by:

$$F_c = \frac{mv^2}{R}$$

Plugging in the given values:

$$F_c = \frac{(1000 \text{ kg})(100 \text{ m/s})^2}{100 \text{ m}} = 1000(100) = 100,000 \text{ N}$$

Expressed to two significant figures in scientific notation, this is $\boxed{1.0 \times 10^5 \text{ N}}$.

GUTS 04.

Resolution Revolution

(a) A simple refracting telescope uses a primary objective lens with a focal length $f = 2.0$ m. If a distant star subtends an angle of $\alpha = 5.0 \times 10^{-5}$ radians in the sky, what is the physical size (linear diameter) of the star's image formed at the focal plane? Express your answer in micrometers (μm) to two significant figures.

(b) To capture the image, a digital sensor is placed at the focal plane. If the objective lens has a diameter of $D = 10$ cm, what is the theoretical minimum angular separation (the diffraction limit) of two stars that this telescope can resolve? Assume the light has a wavelength of $\lambda = 550$ nm and use the Rayleigh criterion. Express your answer in microradians (μrad) to two significant figures.

(c) Consider the intensity of the light. If the telescope is pointed at a source that provides a uniform intensity I_0 at the objective lens, and an eyepiece is added to magnify the image by a factor of $M = 20$, the light is concentrated into a smaller area (the exit pupil). Neglecting any transmission losses, what is the ratio of the light intensity at the exit pupil to the intensity I_0 at the objective? Express your answer to two decimal places.

Answer

(a) 1.0×10^2

(b) 6.7

(c) 400.00

Solution

(a) The physical size d of the image at the focal plane is given by the product of the focal length f and the angular size α in radians:

$$d = f\alpha$$

Plugging in the given values:

$$d = (2.0 \text{ m})(5.0 \times 10^{-5} \text{ rad}) = 1.0 \times 10^{-4} \text{ m}$$

Converting to micrometers, we get 100 μm. To two significant figures, this is 1.0×10^2 μm.

(b) The diffraction limit is determined by the Rayleigh criterion:

$$\theta = 1.22\frac{\lambda}{D}$$

Substituting the given values:

$$\theta = 1.22\left(\frac{550 \times 10^{-9} \text{ m}}{0.10 \text{ m}}\right) = 6.71 \times 10^{-6} \text{ rad}$$

This is 6.71 μrad. Rounding to two significant figures yields $\boxed{6.7\ \mu\text{rad}}$.

(c) By conservation of energy (neglecting transmission losses), the total power collected by the objective lens equals the power exiting the eyepiece. The power is $P = I_0 A_{\text{obj}}$. The magnification of a telescope is the ratio of the objective diameter to the exit pupil diameter, $M = D_{\text{obj}}/D_{\text{exit}}$. Therefore, the area of the exit pupil is $A_{\text{exit}} = A_{\text{obj}}/M^2$. The intensity at the exit pupil is the power divided by this new area:

$$I_{\text{exit}} = \frac{P}{A_{\text{exit}}} = \frac{I_0 A_{\text{obj}}}{(A_{\text{obj}}/M^2)} = I_0 M^2$$

The ratio of the intensities is simply M^2.

$$\frac{I_{\text{exit}}}{I_0} = 20^2 = 400$$

Expressing to two decimal places, we get $\boxed{400.00}$.

GUTS 05.

Final Depth Speedrun (Any% No Buoyancy)

(a) A research submersible descends to the wreck of the Titanic at a depth of 3800 m. The seawater density increases linearly with depth according to $\rho(z) = \rho_0(1+\beta z)$, where $\rho_0 = 1025$ kg/m^3 and $\beta = 1\times10^{-4}$ m^{-1}. Assume that gravity $g = 9.8$ m/s^2. Ignoring atmospheric pressure, what is the pressure at 3800 m? Give your answer in MPa, to two significant figures.

(b) Inside the submersible, an air-filled tank is used for buoyancy control. Initially the submersible is at equilibrium with the surface water. To submerge, the submersible's tank is filled with surface water up to a certain amount, which determines how deep the submersible goes. The submersible has a total volume of $V = 1000$ m^3, including the volume of the tank, and 333 m^3 of surface water are let into the tank. Ignore the mass of air that leaves. Determine the final depth, in meters to two significant figures, of the submersible before it stops sinking due to buoyancy.

(c) Thermal contraction also matters. Model the submersible as a cube with side length 10 m in all directions, made of a material with a linear thermal expansion coefficient $\alpha = 8 \times 10^{-5}\,{}^\circ\text{C}^{-1}$. In a fluid with $\rho_0 = 1025\,\text{kg}\,\text{m}^{-3}$, if the temperature drops by 10 °C, what is the magnitude of the change in buoyancy force, in Newtons to 2 significant figures?

Answer

(a) 45

(b) 3300

(c) 2.4 $\times 10^4$

Solution

(a)

$$\begin{aligned} dP &= \rho(z) g\, dz \\ P &= 9.8 \times 1025 \int_0^{3800} (1 + 1 \times 10^{-4} z)\, dz \\ &= 10045 \times \left[z + \frac{10^{-4}}{2} z^2 \right]_0^{3800} \\ &= 10045 \times (3800 + 722) = 45{,}423{,}490 \text{ Pa} \approx 45 \text{ MPa} \end{aligned}$$

(b)

$$\begin{aligned} F_B &= W \\ \rho(z) g V &= \rho_0 g V + \rho_0 g (333) \\ \rho_0 (1 + \beta z)(1000) &= \rho_0 (1000 + 333) \\ 1000(1 + 10^{-4} z) &= 1333 \\ 10^{-4} z &= 0.333 \\ z &= 3330 \text{ m} \approx 3300 \text{ m} \end{aligned}$$

(c)

$$\begin{aligned} \Delta V &= 3\alpha V \Delta T \\ &= 3(8 \times 10^{-5})(1000)(-10) \\ &= -2.4 \text{ m}^3 \\ \Delta F_B &= \rho_0 g \Delta V \\ &= 1025(9.8)(-2.4) \\ &= -24{,}108 \text{ N} \approx -2.4 \times 10^4 \text{ N} \end{aligned}$$

Taking the magnitude yields $\boxed{2.4 \times 10^4 \text{ N}}$.

GUTS 06.

Kirchhoff Final Boss

(a) A parallel-plate capacitor has square plates of side length 5.0 cm separated by 1.0 mm. A dielectric slab with dielectric constant $\kappa = 4.0$ is partially inserted between the plates and completely fills the gap. The capacitor is connected to a 200 V battery. Ignoring fringing effects, determine the magnitude of the force pulling the dielectric farther into the capacitor, in Newtons to two significant figures.

(b) Find V_1 in Figure 1 in volts to two significant figures.

(c) For the AC circuit in Figure 2, determine the magnitude of the total impedance in ohms to two significant figures.

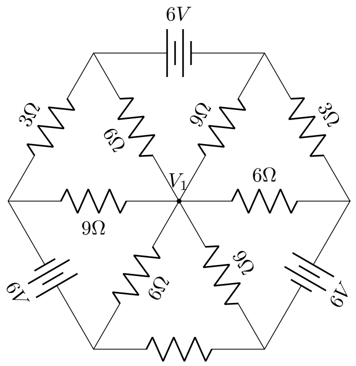


Figure 1

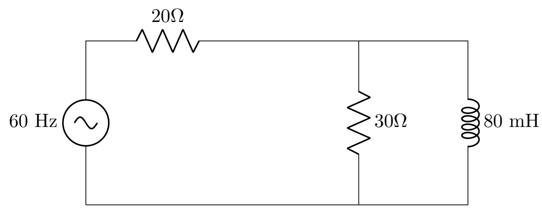


Figure 2

Solution

(a)

$$\begin{aligned} C(x) &= \frac{\epsilon_0 L(L-x)}{d} + \frac{\kappa\epsilon_0 L x}{d} \\ &= \frac{\epsilon_0 L}{d}\left(L + (\kappa-1)x\right) \\ F &= \frac{1}{2}V^2\frac{dC}{dx} \\ &= \frac{1}{2}V^2\frac{\epsilon_0 L(\kappa-1)}{d} \\ &= \frac{1}{2}(200)^2\frac{(8.85\times10^{-12})(0.050)(3.0)}{1.0\times10^{-3}} \\ &= 2.7\times10^{-5}\text{ N} \end{aligned}$$

(b) Let the three outer nodes connected to the center by 9Ω resistors be the reference nodes at 0 V. By symmetry, those three nodes have the same potential, and the other three outer nodes also have a common potential. Each battery raises the potential by 6 V, so the nodes connected to the center by 6Ω resistors are all at 6 V.

Apply KCL at the center node:

$$\begin{aligned} 3\frac{V_1-6}{6} + 3\frac{V_1}{9} &= 0 \\ \frac{V_1-6}{2} + \frac{V_1}{3} &= 0 \\ 3(V_1-6) + 2V_1 &= 0 \\ 5V_1 &= 18 \end{aligned}$$

Therefore

$$V_1 = \frac{18}{5} = 3.6\text{ V}$$

Since the circuit has no explicit ground, only voltage differences matter; this is the value relative to the symmetric 0 V choice above.

(c)

$$\begin{aligned} X_L &= \omega L \\ &= 2\pi f L \\ &= 2\pi(60)(0.080) \\ &= 30.2\Omega \\ Z_L &= i30.2\Omega \\ Z_{\text{parallel}} &= \frac{(30)(i30.2)}{30 + i30.2} \\ &= 15.1 + 15.0j \\ Z_{\text{tot}} &= 20 + Z_{\text{parallel}} \\ &= 35.1 + 15.0i \\ |Z_{\text{tot}}| &= \sqrt{(35.1)^2 + (15.0)^2} \\ &= 38\Omega \end{aligned}$$

Answer

(a) 2.7×10^{-5} N

(b) 3.6

(c) 38Ω

GUTS 07.

Relative to Super-Humans

Stanley and Ashton are floating in space.

(a) Ashton throws a baseball at 80% the speed of light, relative to some frame S). After some time, Stanley flies after it at 95% the speed of light relative to S. What's the velocity of the ball in Stanley's frame as he is chasing after it? The ball and Stanley are traveling the same, positive, direction in S. Express your answer in terms of c.

(b) Now Stanley and Ashton are racing. At a certain time relative to a reference frame S, Stanley is at point $x = 0$ m and Ashton is at $x = 10$ m. Stanley flies at a constant $0.90c$ and Ashton flies at a constant $0.85c$. According to Stanley, what is the time between when he arrived at point $x = 0$ in frame S and when Ashton reached $x = 10$ in frame S?

(c) Now Stanley and Ashton are in a throwing competition. Ashton throws a cylindrical rod straight along its axis of symmetry at 80% the speed of light relative to Stanley's frame of reference. The rod has proper length $L_0 = 4.0$ m, proper radius $r_0 = 0.5$ m, and rest mass $m = 20.0$ kg. What is the density of the rod in Stanley's reference frame, in kilograms per cubic meter to three significant figures?

Solution

(a) Use the relativistic velocity transformation:

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

Here $u = 0.80c$ is the baseball speed in S, and Stanley moves at $v = 0.95c$. Thus

$$u' = \frac{0.80c - 0.95c}{1 - (0.80)(0.95)} = \frac{-0.15c}{0.24} = -0.625c$$

Therefore

$$\boxed{u' = -0.625c}$$

(b) Consider the two events defined in frame S: Stanley is at $x = 0$ and Ashton is at $x = 10$ m at the same time $t = 0$. Stanley's Lorentz factor is

$$\gamma = \frac{1}{\sqrt{1 - 0.90^2}} \approx 2.294$$

Transform Ashton's event into Stanley's frame:

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) = 2.294\left(0 - \frac{(0.90c)(10)}{c^2}\right) \approx -6.9 \times 10^{-8} \text{ s}$$

The negative sign means Ashton reaches $x = 10$ m earlier than Stanley reaches $x = 0$ in Stanley's frame, so the time difference is

$$\boxed{6.9 \times 10^{-8} \text{ s}}$$

with Ashton's event first.

(c) For $v = 0.80c$,

$$\gamma = \frac{1}{\sqrt{1 - 0.80^2}} = \frac{5}{3}$$

The rod's proper volume is

$$V_0 = \pi r_0^2 L_0 = \pi(0.5)^2(4.0) = \pi \text{ m}^3$$

Only the length contracts, so the volume in Stanley's frame is

$$V = \frac{V_0}{\gamma}$$

Hence the density is

$$\rho = \frac{m}{V} = \gamma\frac{m}{V_0} = \frac{5}{3} \cdot \frac{20.0}{\pi} = \frac{100}{3\pi} \text{ kg/m}^3 \approx 10.6 \text{ kg/m}^3$$

Therefore

$$\boxed{10.6 \text{ kg/m}^3}$$

Answer

(a) $-0.625c$

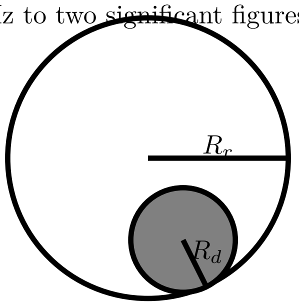
(b) 6.9×10^{-8} s, with Ashton's event earlier

(c) 10.6 kg/m^3

GUTS 08.

Oscillations, Oscillations, Oscillations

(a) All motion in this problem is on one plane. A disk with radius $R_d = 1$m of uniform density is contained in a ring of radius $R_r = 3$m. Both have the same mass of $m = 2$kg. The ring is free to rotate around its center, but its center is fixed from moving. The disk rolls around on the bottom of the ring without slipping. If the disk is released from rest slightly away from the bottom, but in contact with the ring, what is the frequency of oscillation in Hz to two significant figures? Take $g = 9.8$.



(b) A vertical pole of length $L = 3.00\text{m}$ is installed on the north pole, normal to the ground. An experimenter manages to set the tether ball attached to the top into a perfectly conical motion of $\theta = \pi/6$ from the vertical. The ball moves around the horizontal circle with constant speed. The earth spins with $\Omega = 7.29 \times 10^{-5}\text{s}^{-1}$. Because of the Coriolis force, the time it takes for the ball to complete a prograde motion is different from retrograde motion. Find $|T_p - T_r|$ in seconds to two significant figures and in scientific notation. Neglect air resistance and treat the ball as a point mass.

(c) Model an exotic particle as an alpha particle, mass $6.64 \times 10^{-27}\text{kg}$ and charge $3.2 \times 10^{-19}\text{C}$, and a proton, charge $1.6 \times 10^{-19}\text{C}$, attached by a string with spring constant $k = 5\text{N/m}$. The spring has a rest length 0. Let the particle be rotating at $\omega = 6 \times 10^{12}\text{s}^{-1}$. Around its equilibrium length given the rotation, what is the frequency of small oscillations in the length of the spring in Hz, to two significant figures in scientific notation? Ignore relativistic and quantum effects.

Answer

(a) 3.1×10^{-1} Hz

(b) 2.4×10^{-4} s

(c) 1.7×10^{13} Hz

Solution

(a) Let $R = R_r - R_d = 2$ m. If θ is the angular displacement of the disk center and ϕ is the ring's rotation angle, the rolling constraint is

$$R\dot{\theta} - R_d\dot{\gamma} = R_r\dot{\phi}$$

The kinetic energy is

$$T = \frac{1}{2}mR^2\dot{\theta}^2 + \frac{1}{4}m(R\dot{\theta} - R_r\dot{\phi})^2 + \frac{1}{2}mR_r^2\dot{\phi}^2,$$

since $I_{\text{disk}} = \frac{1}{2}mR_d^2$ and $I_{\text{ring}} = mR_r^2$.

Because ϕ is cyclic, its conjugate momentum is conserved. The system starts from rest, so that conserved quantity is zero:

$$-\frac{1}{2}mRR_r\dot{\theta} + \frac{3}{4}mR_r^2\dot{\phi} = 0 \quad \Rightarrow \quad \dot{\phi} = \frac{R}{3R_r}\dot{\theta} = \frac{2}{9}\dot{\theta}$$

Substituting back gives

$$T = \frac{2}{3}mR^2\dot{\theta}^2 = \frac{1}{2}I_{\text{eff}}\dot{\theta}^2, \qquad I_{\text{eff}} = \frac{4}{3}mR^2$$

The potential energy is

$$V = -mgR\cos\theta \approx \text{const} + \frac{1}{2}mgR\,\theta^2$$

Therefore the small-oscillation angular frequency is

$$\omega^2 = \frac{mgR}{I_{\text{eff}}} = \frac{3g}{4R} = \frac{3(9.8)}{8} = 3.675,$$

so

$$\omega \approx 1.92\ \text{s}^{-1}, \qquad f = \frac{\omega}{2\pi} \approx 0.305\ \text{Hz}$$

Hence

$$\boxed{f \approx 3.1 \times 10^{-1}\ \text{Hz}}$$

(b) For an ordinary conical pendulum,

$$\omega_0^2 = \frac{g\tan\theta}{L\sin\theta} = \frac{g}{L\cos\theta}$$

At the north pole, the Coriolis force modifies the radial force balance to

$$\omega^2 + 2\Omega\omega - \omega_0^2 = 0$$

Thus the prograde and retrograde angular speeds are

$$\omega_p = \sqrt{\Omega^2 + \omega_0^2} - \Omega, \qquad |\omega_r| = \sqrt{\Omega^2 + \omega_0^2} + \Omega$$

Their periods differ by

$$|T_p - T_r| = 2\pi \left| \frac{1}{\omega_p} - \frac{1}{|\omega_r|} \right|$$
$$= \frac{4\pi\Omega}{\omega_0^2} = \frac{4\pi\Omega L\cos\theta}{g}$$

Substituting $L = 3.00$ m, $\theta = \pi/6$, $\Omega = 7.29 \times 10^{-5}\ \mathrm{s}^{-1}$, and $g = 9.8\ \mathrm{m/s^2}$ gives

$$|T_p - T_r| = \frac{4\pi(7.29 \times 10^{-5})(3.00)\cos(\pi/6)}{9.8} \approx 2.4 \times 10^{-4}\ \mathrm{s}$$

(c) Let μ be the reduced mass:

$$\mu = \frac{m_\alpha m_p}{m_\alpha + m_p} \approx 1.33 \times 10^{-27}\ \mathrm{kg}$$

For the relative coordinate r, the effective potential is

$$U_{\text{eff}}(r) = \frac{L^2}{2\mu r^2} + \frac{1}{2}kr^2 + \frac{q_1 q_2}{4\pi\varepsilon_0 r}$$

At the circular equilibrium, $L = \mu r_e^2 \omega$, so

$$\frac{q_1 q_2}{4\pi\varepsilon_0 r_e^3} = k - \mu\omega^2$$

The small radial oscillation angular frequency satisfies

$$\Omega_r^2 = \frac{1}{\mu} U''_{\text{eff}}(r_e)$$

Differentiating twice gives

$$U''_{\text{eff}}(r_e) = 3\mu\omega^2 + k + 2\left(k - \mu\omega^2\right) = 3k + \mu\omega^2,$$

so

$$\Omega_r = \sqrt{\frac{3k}{\mu} + \omega^2}$$

With $k = 5$ N/m and $\omega = 6 \times 10^{12}\ \mathrm{s}^{-1}$,

$$\Omega_r \approx 1.06 \times 10^{14}\ \mathrm{s}^{-1}$$

The frequency in hertz is

$$f = \frac{\Omega_r}{2\pi} \approx 1.7 \times 10^{13}\ \mathrm{Hz}$$

Thus

$$\boxed{1.7 \times 10^{13}\ \mathrm{Hz}}$$

GUTS 09.

Enrico.

Fermi Questions! Estimate all values, you will get credit on how close to the correct answer. Submit every answer using scientific notation, such as 1.23×10^4. Your submission is capped to 3 significant figures.

(a) Berkeley is famous for its high density of boba shops. If every boba pearl served in the city of Berkeley in a single day were lined up end-to-end, how many times could they wrap around the perimeter of the Campanile (our bell tower)?

(b) The Bevatron at Lawrence Berkeley National Lab, where the antiproton was discovered, used about 10,000 tons of steel for its magnets. If you scrapped all that metal to replace the perpetually broken washing machines in the Berkeley dorms, how many units could you actually build?

(c) Many Berkeley students suffer in electronics lab classes, and often go insane. How many alkaline AAA batteries would it take one of these students to charge a Tesla Model 3?

Solution

(a) Estimate about 20 boba shops in Berkeley and about 300 drinks per shop per day:

$$20 \cdot 300 \approx 6 \times 10^3 \text{ drinks/day}$$

If each drink contains about 70 pearls of diameter 8×10^{-3} m, each drink contributes

$$70(8 \times 10^{-3}) \approx 5.6 \times 10^{-1} \text{ m}$$

of pearl length. The total daily length is then

$$(6 \times 10^3)(5.6 \times 10^{-1}) \approx 3.4 \times 10^3 \text{ m}$$

Taking the Campanile perimeter to be about 60 m,

$$\frac{3.4 \times 10^3}{60} \approx 5.6 \times 10^1$$

This gives

$$\boxed{6.00 \times 10^1}$$

(b) 10,000 tons of steel is about

$$10^4 \cdot 10^3 = 10^7 \text{ kg}$$

of steel. If a washing machine uses roughly 70 kg of steel, then

$$\frac{10^7}{70} \approx 1.4 \times 10^5$$

This gives

$$\boxed{1.40 \times 10^5}$$

(c) A Tesla Model 3 battery is roughly 60 kWh $= 6.0 \times 10^4$ Wh. An alkaline AAA battery stores about 1.8 Wh. Thus

$$\frac{6.0 \times 10^4}{1.8} \approx 3.3 \times 10^4$$

This gives

$$\boxed{3.30 \times 10^4}$$

Answer

(a) 6.00×10^1

(b) 1.40×10^5

(c) 3.30×10^4